Exam Revision on Indices

Use your CAS calculator to check your answers.

Rules of Indices: $a^m \times a^n = a^{m+n}$ $\quad a^m \div a^n = a^{m-n}$ $\quad \left(a^m\right)^n = a^{mn}$

Negative and fractional indices: $a^{-n} = \dfrac{1}{a^n}$ $\quad a^{\frac{1}{n}} = \sqrt[n]{a}$ $\quad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ *or* $\sqrt[n]{a^m}$

Special indices: $a^1 = a$ and $a^0 = 1$

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1. Simplify the following expressions:

a. $x^4 \times x^3$ b. $y^5 \times y^2$ c. $p^3 \times p$ d. $x^7 \times x^{-3}$

e. $x^5 \div x^2$ f. $\dfrac{y^7}{y^3}$ g. $n^{10} \div n^4$ h. $z^6 \div z^3$

i. $x^5 \div x^{-2}$ j. $\dfrac{w^5}{w^{-3}}$ k. $p^5 \div p^{-2}$ l. $x^3 \div x^{-3}$

m. $\left(x^3\right)^2$ n. $\left(y^3\right)^{-2}$ o. $\left(z^{-1}\right)^{-3}$ p. $\left(\dfrac{1}{w^2}\right)^{-3}$

2. Simplify the following expressions:

a. $(ab)^4$ b. $(xy)^3$ c. $\dfrac{3}{(3a)^2}$ d. $(2a)^3$

e. $(3y)^4$ f. $\left(mn^2\right)^5$

3. Simplify the following expressions:

a. $2a^2 \times a^3$ b. $x^2 \times 3x^4$ c. $2y^2 \times 3y^2$ d. $8x^2 \div 2x$

e. $10y^3 \div 2y^2$ f. $6z^4 \div 3z^2$ g. $10y^2 \div 5y^3$ h. $12x^4 \div 3x^{-2}$

4. Multiply out the brackets:

a. $x^3\left(x^2 + x^4\right)$ b. $y^2\left(y^3 - y\right)$ c. $z^2\left(z^4 - 1\right)$

5. Simplify:

a. $\dfrac{x^3 \times x^4}{x^2}$ b. $\dfrac{t^3 \times t^6}{t^4}$ c. $\dfrac{w^2 \times w^{-3}}{w^{-4}}$

6. Solve the equation for n:

a. $2^n = 8$ b. $3^n = 81$ c. $4^n = 4$ d. $3^n = \frac{1}{3}$

7. Simplify and express with positive indices:

a. $3x^{-1}$ b. $5y^{-3}$ c. $\frac{1}{2}x^{-1}$ d. $\frac{3}{4}u^{-3}$

8. Write in root form:

a. $m^{\frac{3}{4}}$ b. $x^{\frac{4}{5}}$ c. $k^{\frac{2}{3}}$ d. $x^{\frac{1}{2}}$

e. $x^{-\frac{1}{2}}$ f. $x^{-\frac{2}{3}}$

9. Write in index form:

a. $\sqrt[3]{y^5}$ b. $\sqrt[3]{z^4}$ c. $\sqrt{x}$ d. $\sqrt[3]{t}$

10. Evaluate:

a. $9^{\frac{1}{2}}$ b. $8^{\frac{1}{3}}$ c. $27^{\frac{1}{3}}$ d. $8^{\frac{2}{3}}$

e. $27^{\frac{2}{3}}$ f. $4^{\frac{3}{2}}$ g. $25^{\frac{3}{2}}$ h. $81^{\frac{3}{4}}$

11. Simplify the following expressions:

a. $u^{\frac{3}{2}} \times u^{\frac{1}{2}}$ b. $v^{\frac{4}{3}} \times v^{-\frac{1}{3}}$ c. $x^{\frac{3}{2}} \div x^{\frac{1}{2}}$ d. $y^{\frac{3}{4}} \div y^{-\frac{1}{4}}$

e. $\left(t^{\frac{1}{2}}\right)^2$ f. $\left(p^{-\frac{1}{2}}\right)^2$ g. $\left(q^{\frac{1}{4}}\right)^0$ h. $\left(r^3\right)^{\frac{1}{3}}$

12. Simplify the following:

a. $x^{\frac{1}{2}}\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)$ b. $m^{-\frac{3}{4}}\left(m^{\frac{7}{4}} - m^{-\frac{1}{4}}\right)$ c. $\frac{a^3 \times a^{-2}}{a}$ d. $\frac{c^2 \times c^{-2}}{c^{-1}}$

e. $\frac{x^{\frac{1}{2}} \times x^{-\frac{3}{2}}}{x^2}$ f. $\frac{y^{-\frac{1}{3}} \times y^{\frac{4}{3}}}{y}$ g. $\left(a^2 + 1\right)\left(a^{-2} + 1\right)$ h. $\left(b^{-1} + 1\right)\left(b^{-1} - 1\right)$

e. $\left(x^{\frac{1}{2}} + 1\right)\left(x^{\frac{1}{2}} + 1\right)$ f. $\left(u^{\frac{1}{2}} + 1\right)\left(u^{-\frac{1}{2}} + 1\right)$

Indices Revision Sheet

1) Simplify the following, expressing your answers with positive indices where necessary:

a) $3a^2b^9 \times 6a^6b^3$ b) $\frac{16b^7}{42b^5}$ c) $\frac{x^8y^{12} \times x^5y^3}{x^2y \times x^3y^2}$ d) $5d^0$ e) $4 - 3(e^2f^5g^9)^0$

f) $(2x^4)^5$ g) $\left(\frac{g^5h^3}{k^2}\right)^6$ h) $\frac{(5ab^2)^2}{3ab^3} \times \frac{(3a^2)^3b^5}{5a^2b^3}$ i) $a^{-3}b^{-5} \times a^5b^{-4}$

j) $\frac{9g^2h^4k^{-5}}{21g^{-3}h^4k^3}$ k) $\left(\frac{p^{-2}q^3}{p^2q}\right)^3 \times \frac{(pq^{-6})^2}{p^3q^{-4}}$ l) $\frac{7(a^{-5}b^4)^{-3}}{(2a^3b^{-2})^{-3}} \div \frac{21(a^{-3}b^2)^{-2}}{(2a^{-4})^5(b^5)^4}$ m) $x^{1/2} \times x^{1/4}$

n) $\frac{(m^{1/3}n^{1/2})^3}{(mn^{-1})^{1/2}} \times \frac{mn^{-1}}{m^{-1/2}n}$ o) $\frac{a^5(b^{-3})^{-1/2}}{3a^2b} \times \left(\frac{ab^{4/3}}{a^{1/3}b}\right)^{-3}$

Solutions:

1a) $18a^8b^{12}$ b) $\frac{8b^2}{21}$ c) x^8y^{12} d) 5 e) 1 f) 2^5x^{20} or $32\,x^{20}$ g) $\frac{g^{30}h^{18}}{k^{12}}$ h) $45a^5b^3$ i) $\frac{a^2}{b^9}$ j) $\frac{3g^5}{7k^8}$

k) $\frac{1}{p^{13}q^2}$ l) $\frac{2^8b^6}{3a^2}$ or $\frac{256b^6}{3a^2}$ m) $x^{3/4}$ n) m^2 o) $\frac{a}{3b^{1/2}}$