Qn 6 a) $(5^2 \times 2)^0 \times (5^{-3} \times 2^0)^5 \div (5^6 \times 2^{-1})^{-3}$

Indices: $a^{-m} = \frac{1}{a^m}$

$= 1 \times (5^{-3} \times 1)^5 \div \frac{1}{(5^6 \times 2^{-1})^3}$

$\div \frac{1}{a} \rightarrow \times \frac{a}{1}$

$= 5^{-15} \times (5^6 \times 2^{-1})^3$

$(a^m)^n = a^{mn}$

$= 5^{-15} \times 5^{18} \times 2^{-3}$

$a^m \times a^n = a^{m+n}$.

$= 5^{-15+18} \times 2^{-3}$

write with positive power.

$= 5^3 \times \frac{1}{2^3}$

$= \frac{125}{8}$

b) $(2^3 \times 3^3)^{-2} \div \frac{(2^6 \times 3^9)^0}{2^6 \times (3^{-2})^{-3}}$

$\div \frac{a}{b} \rightarrow \times \frac{b}{a}$

$= 2^{-6} \times 3^{-6} \times \frac{2^6 \times 3^6}{(2^6 \times 3^9)^0}$

$a^0 = 1$

$= \frac{2^{-6} \times 3^{-6} \times 2^6 \times 3^6}{1}$

Multiply $a^m \times a^n = a^{m+n}$.

$= 2^{-6+6} \times 3^{-6+6}$

$= 2^0 \times 3^0$

$= 1 \times 1$

$= 1$

Qn 7. $(2x)^{-3} \times \left(\frac{x^2}{2}\right)^2 \div \frac{2x}{(2^3)^4}$

Indices: $(a^m)^n$ and $\div \frac{a}{b} \Rightarrow \times \frac{b}{a}$.

$= 2^{-3} x^{-3} \times \frac{x^2}{2^2} \times \frac{2^{12}}{2x}$

Multiply terms $a^m \times a^n = a^{m+n}$

$= \frac{2^{-3+12} \times x^{-3+2}}{2^{2+1} \times x}$

simplify

$= \frac{2^9 \times x^{-1}}{2^3 \times x}$

Divide terms $\frac{a^m}{a^n} = a^{m-n}$.

$= 2^{9-3} \times x^{-1-1}$

simplify

$= 2^6 \times x^{-2}$

write with positive powers.

$= 2^6 \times \frac{1}{x^2}$

$= \frac{2^6}{x^2}$

When $x = 8$, $x = 2^3$

$\therefore \frac{2^6}{x^2} = \frac{2^6}{(2^3)^2}$

$= \frac{2^6}{2^6}$

$= 1$.

Qn 8

$$\frac{a^{2y} \times 9b^{y} \times (5ab)^{y}}{(a^{y})^{3} \times 5(3b^{y})^{2}}$$

Indices $(a^m)^n = a^{mn}$.

$$= \frac{a^{2y} \times 9b^{y} \times 5^{y}a^{y}b^{y}}{a^{3y} \times 5 \times 3^{2}b^{2y}}$$

Collect like terms.

$$= \frac{\cancel{9^{1}} \times \cancel{5^{y}} \times a^{2y} \times a^{y} \times b^{y} \times b^{y}}{\cancel{5} \times \cancel{3^{2}} \times a^{3y} \times b^{2y}}$$

Multiply $[a^m \times a^n = a^{m+n}]$
- in numerator
- in denominator.

$$= \frac{5^{y} \times a^{2y+y} \times b^{y+y}}{5 \times a^{3y} \times b^{2y}}$$

Simplify.

$$= \frac{5^{y} \times a^{3y} \times b^{2y}}{5^{1} \times a^{3y} \times b^{2y}}$$

Divide $\frac{a^m}{a^n} = a^{m-n}$.

$$= 5^{y-1} \times a^{3y-3y} \times b^{2y-2y}$$

$$= 5^{y-1} \times a^{0} \times b^{0}$$

$$= 5^{y-1}$$

If $5^{y-1} = 125$
$= 5^{3}$ → write both sides with same base.

since $5^{y-1} = 5^{3}$ → powers are the same value.

$y - 1 = 3$ → +1 to both sides.

$y = 4$

Qn 9 (A) $(4xy)^{\frac{3}{2}}$

$= 4^{\frac{3}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}$

$= (2^2)^{\frac{3}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}$

$= 2^{2 \times \frac{3}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}$

$= 2^3 x^{\frac{3}{2}} y^{\frac{3}{2}}$

$= 8 x^{\frac{3}{2}} y^{\frac{3}{2}}$.

∴ A is correct.

(B) $(4xy)^{\frac{3}{2}}$

$= (4xy)^{3 \times \frac{1}{2}}$

$= (4xy)^{\frac{1}{2} \times 3}$

$= [(4xy)^{\frac{1}{2}}]^3$) $a^{\frac{1}{2}} = \sqrt{a}$.

$= [\sqrt{4xy}]^3$

∴ B is correct.

(C) $(4xy)^{\frac{3}{2}}$

$= (4xy)^{3 \times \frac{1}{2}}$

$= [(4xy)^3]^{\frac{1}{2}}$

$(a^m)^n = a^{mn}$

$= [4^3 x^3 y^3]^{\frac{1}{2}}$

$= \sqrt{4^3 x^3 y^3}$

$= \sqrt{64 x^3 y^3}$

∴ C is correct.

(D) $\frac{(2x^3y^3)^{\frac{1}{2}}}{(\sqrt{32})^{-1}}$

$= \frac{2^{\frac{1}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}}{(\sqrt{2^5})^{-1}}$

$= \frac{2^{\frac{1}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}}{[(2^5)^{\frac{1}{2}}]^{-1}}$

$= \frac{2^{\frac{1}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}}{2^{5 \times \frac{1}{2} \times -1}}$

$= \frac{2^{\frac{1}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}}{2^{-\frac{5}{2}}}$

$= 2^{\frac{1}{2} - (-\frac{5}{2})} x^{\frac{3}{2}} y^{\frac{3}{2}}$

$= 2^{\frac{6}{2}} x^{\frac{3}{2}} y^{\frac{3}{2}}$

$= (2^2)^{\frac{3}{2}} \times x^{\frac{3}{2}} \times y^{\frac{3}{2}}$ ∴ D is correct.

∴ E is incorrect.

10. $\dfrac{x^2y}{(2xy^2)^3} \div \dfrac{xy}{16x^0}$

Indices $(a^m)^n = a^{mn}$ and $\div \frac{a}{b} \rightarrow \times \frac{b}{a}$

$= \dfrac{x^2y}{2^3x^3y^6} \times \dfrac{16x^0}{xy}$

$= \dfrac{x^2y}{2^3x^3y^6} \times \dfrac{2^4 \times 1}{xy}$

$= \dfrac{2^4}{2^3} \times \dfrac{x^2}{x^3 \times x} \times \dfrac{y}{y^6 \times y}$

$= 2 \times x^{2-4} \times y^{1-7}$

$= 2 \times x^{-2} \times y^{-6}$

$= 2 \times \dfrac{1}{x^2} \times \dfrac{1}{y^6}$

$= \dfrac{2}{x^2y^6}$.

∴ (A) is correct.

Qn 11. a) $\sqrt[3]{m^2n} \div \sqrt{mn^3}$

$= (m^2n)^{\frac{1}{3}} \div (mn^3)^{\frac{1}{2}}$

$= m^{\frac{2}{3}} n^{\frac{1}{3}} \div m^{\frac{1}{2}} n^{\frac{3}{2}}$

$= \dfrac{m^{\frac{2}{3}} n^{\frac{1}{3}}}{m^{\frac{1}{2}} n^{\frac{3}{2}}}$

$= m^{\frac{2}{3}-\frac{1}{2}} \; n^{\frac{1}{3}-\frac{3}{2}}$

$= m^{\frac{4}{6}-\frac{3}{6}} \; n^{\frac{2}{6}-\frac{9}{6}}$

$= m^{+\frac{1}{6}} \; n^{-\frac{7}{6}}$ $\Rightarrow$ OR $(mn^{-7})^{\frac{1}{6}} \cdot \sqrt[6]{mn^{-7}}$

$= m^{\frac{1}{6}} \times \dfrac{1}{n^{\frac{7}{6}}}$ $= \sqrt[6]{m \times \dfrac{1}{n^7}}$

$= \sqrt[6]{\dfrac{m}{n^7}}$

b) $\left(g^{-2}h\right)^3 \times \left(\dfrac{1}{n^{-3}}\right)^{\frac{1}{2}}$

$= g^{-6} \times h^3 \times \left[\left(n^{-3}\right)^{-1}\right]^{\frac{1}{2}}$

$= g^{-6} \times h^3 \times \left[n^3\right]^{\frac{1}{2}}$

$= \dfrac{1}{g^6} \times h^3 \times \sqrt{n^3}$

$= \dfrac{h^3\sqrt{n^3}}{g^6}$

11 B c) $\dfrac{45^{\frac{1}{3}}}{9^{\frac{3}{4}} \times 15^{\frac{3}{2}}} = \dfrac{(9 \times 5)^{\frac{1}{3}}}{9^{\frac{3}{4}} \times (5 \times 3)^{\frac{3}{2}}}$ Write with base 3 and 5.

$= \dfrac{(3^2 \times 5)^{\frac{1}{3}}}{(3^2)^{\frac{3}{4}} \times 5^{\frac{3}{2}} \times 3^{\frac{3}{2}}}$

$= \dfrac{3^{2 \times \frac{1}{3}} \times 5^{\frac{1}{3}}}{3^{2 \times \frac{3}{4}} \times 5^{\frac{3}{2}} \times 3^{\frac{3}{2}}}$

$= \dfrac{3^{\frac{2}{3}} \times 5^{\frac{1}{3}}}{3^{\frac{3}{2}} \times 5^{\frac{3}{2}} \times 3^{\frac{3}{2}}}$ ← oops!

$= \dfrac{3^{\frac{2}{3}} \times 5^{\frac{1}{3}}}{3^{\frac{3}{2}+\frac{3}{2}} \times 5^{\frac{3}{2}}}$

$= \dfrac{3^{\frac{2}{3}} \times 5^{\frac{1}{3}}}{3^{\frac{6}{2}} \times 5^{\frac{3}{2}}}$

$= 3^{\frac{2}{3}-\frac{6}{2}} \times 5^{\frac{1}{3}-\frac{3}{2}}$

$= 3^{\frac{4}{6}-\frac{18}{6}} \times 5^{\frac{2}{6}-\frac{9}{6}}$

$= 3^{-\frac{14}{6}} \times 5^{-\frac{7}{6}}$

$= 3^{-\frac{7}{3}} \times 5^{-\frac{7}{6}}$

11 d) $2^{\frac{3}{2}} \times 4^{-\frac{1}{4}} \times 16^{-\frac{3}{4}}$

$= 2^{\frac{3}{2}} \times (2^2)^{-\frac{1}{4}} \times (2^4)^{-\frac{3}{4}}$

$= 2^{\frac{3}{2}} \times 2^{-\frac{2}{4}} \times 2^{-\frac{12}{4}}$

$= 2^{\frac{3}{2}} \times 2^{-\frac{1}{2}} \times 2^{-3}$

$= 2^{\frac{3}{2} + (-\frac{1}{2}) - 3}$

$= 2^{\frac{3}{2} - \frac{1}{2} - \frac{6}{2}}$

$= 2^{-\frac{4}{2}}$

$= 2^{-2}$

e) $\left(\frac{a^3 b^{-2}}{3^{-3} b^{-3}}\right)^{-2} \div \left(\frac{3^{-3} a^{-2} b}{a^4 b^{-2}}\right)^2$

$= \left(\frac{a^3 b^{-2}}{3^{-3} b^{-3}}\right)^{-2} \times \left(\frac{a^4 b^{-2}}{3^{-3} a^{-2} b}\right)^2$

$= \frac{a^{-6} b^4}{3^6 b^6} \times \frac{a^8 b^{-4}}{3^{-6} a^{-4} b^2}$

$= \frac{a^{-6+4} \times b^{4+(-4)}}{3^{6+(-6)} \times a^{-4} \times b^{6+2}}$

$= \frac{a^{-2} \times b^0}{3^0 \times a^{-4} \times b^8}$ $\quad 1$ $\quad 1$

$= a^{-2-(-4)} \times \frac{1}{b^8}$

$= a^2 \times \frac{1}{b^8} = \frac{a^2}{b^8}$

11 f) $\left(\sqrt[5]{d^2}\right)^{\frac{3}{2}} \times \left(\sqrt[3]{d^5}\right)^{\frac{1}{5}}$

$= \left((d^2)^{\frac{1}{5}}\right)^{\frac{3}{2}} \times \left((d^5)^{\frac{1}{3}}\right)^{\frac{1}{5}}$

$= d^{2 \times \frac{1}{5} \times \frac{3}{2}} \times d^{5 \times \frac{1}{3} \times \frac{1}{5}}$

$= d^{\frac{6}{10}} \times d^{\frac{5}{15}}$

$= d^{\frac{3}{5}} \times d^{\frac{1}{3}}$

$= d^{\frac{3}{5} + \frac{1}{3}}$

$= d^{\frac{9}{15} + \frac{5}{15}}$

$= d^{\frac{14}{15}}$.

12 a) $P = P_0 \times 10^{kt}$

initial no of koalas is 20 $\therefore$ when $t = 0$, $P = 20$

$20 = P_0 \times 10^{k \times 0}$

$20 = P_0 \times 10^0$

$20 = P_0 \times 1$

$20 = P_0$

population after 1 year is 40 $\therefore$ when $t = 1$, $P = 40$

$40 = P_0 \times 10^{k(1)}$

$40 = 20 \times 10^k$

$2 = 10^k$ ← use CAS to solve.