

$$\begin{aligned}
 c) \quad \left( \frac{4a^{\frac{1}{3}}}{b^3} \right)^{\frac{1}{2}} &= \left( \frac{2^2 a^{\frac{1}{3}}}{b^3} \right)^{\frac{1}{2}} \\
 &= \frac{2^{2 \times \frac{1}{2}} \times a^{\frac{1}{3} \times \frac{1}{2}}}{b^{3 \times \frac{1}{2}}} \\
 &= \frac{2 \times a^{\frac{1}{6}}}{b^{\frac{3}{2}}}
 \end{aligned}$$

$$(a^m)^n = a^{mn}$$

simplify powers.

$$\textcircled{15} \ a) \quad \frac{16^{\frac{3}{4}} \times 81^{\frac{1}{4}}}{6 \times 16^{\frac{1}{2}}} \quad \begin{matrix} 16 = 4^2 \\ 16 = 2^4 \end{matrix}$$

$$= \frac{(2^4)^{\frac{3}{4}} \times (3^4)^{\frac{1}{4}}}{2 \times 3 \times (4^2)^{\frac{1}{2}}}$$

$$(a^m)^n = a^{mn}$$

$$= \frac{2^{4 \times \frac{3}{4}} \times 3^{4 \times \frac{1}{4}}}{2 \times 3 \times 4^{2 \times \frac{1}{2}}}$$

simplify powers.

$$= \frac{2^3 \times 3^1}{2 \times 3 \times 4}$$

$$= \frac{8 \times 3}{2 \times 3 \times 4}$$

$$= \frac{24}{24}$$

$$= 1$$

$$b) \left( 125^{\frac{2}{3}} - 27^{\frac{2}{3}} \right)^{\frac{1}{2}}$$

$$= \left( (5^3)^{\frac{2}{3}} - (3^3)^{\frac{2}{3}} \right)^{\frac{1}{2}}$$

$$= \left( 5^{3 \times \frac{2}{3}} - 3^{3 \times \frac{2}{3}} \right)^{\frac{1}{2}}$$

$$= \left( 5^2 - 3^2 \right)^{\frac{1}{2}}$$

$$= (25 - 9)^{\frac{1}{2}}$$

$$= (16)^{\frac{1}{2}}$$

$$= (4^2)^{\frac{1}{2}}$$

$$= 4^{2 \times \frac{1}{2}}$$

$$= 4.$$

$$125 = 5^3$$

$$27 = 3^3$$

$$(a^m)^n = a^{mn}$$

simplify powers.

simplify.

$$(16) a) \sqrt[3]{a^9} + \sqrt[4]{16a^8b^2} - 3 \cdot (\sqrt[5]{a})^{15}$$

$$= (a^9)^{\frac{1}{3}} + (2^4 a^8 b^2)^{\frac{1}{4}} - 3 (a^{\frac{1}{5}})^{\frac{1}{5} \cdot 15}$$

$$= a^{9 \times \frac{1}{3}} + 2^{4 \times \frac{1}{4}} \times a^{8 \times \frac{1}{4}} \times b^{2 \times \frac{1}{4}} - 3 \times a^{\frac{1}{5} \times 15}$$

$$= a^3 + 2 \times a^2 \times b^{\frac{1}{2}} - 3 \times a^3$$

$$= a^3 + 2a^2b^{\frac{1}{2}} - 3a^3$$

$$= -2a^3 + 2a^2b^{\frac{1}{2}}$$

$$\sqrt[m]{a} = a^{\frac{1}{m}}$$

$$(a^m)^n = a^{mn}$$

simplify powers.

simplify terms

collect like terms:  $a^3 - 3a^3 = -2a^3$ .

$$b) \sqrt[5]{32x^5y^{10}} + \sqrt[3]{64x^3y^6}$$

$$= (2^5x^5y^{10})^{\frac{1}{5}} + (2^6x^3y^6)^{\frac{1}{3}}$$

$$= 2^{5 \times \frac{1}{5}} \times x^{5 \times \frac{1}{5}} \times y^{10 \times \frac{1}{5}} + 2^{6 \times \frac{1}{3}} \times x^{3 \times \frac{1}{3}} \times y^{6 \times \frac{1}{3}}$$

$$= 2^1 \times x^1 \times y^2 + 2^2 \times x^1 \times y^2$$

$$= 2xy^2 + 4xy^2$$

$$= 6xy^2$$

$$\sqrt[m]{a} = a^{\frac{1}{m}}$$

$$(a^m)^n = a^{mn}$$

simplify powers

collect like terms

$$(17) a) \frac{(5a^{-2}b)^{-3} \times 4a^6b^{-2}}{2a^2b^3 \times 5^{-2}a^{-3}b^{-6}}$$

$$= \frac{5^{1 \times -3} \times a^{-2 \times -3} \times b^{1 \times -3} \times 2^2 \times a^6 \times b^{-2}}{2a^2b^3 \times 5^{-2}a^{-3}b^{-6}}$$

$$= \frac{5^{-3} \times a^6 \times b^{-3} \times 2^2 \times a^6 \times b^{-2}}{2a^2b^3 \times 5^{-2}a^{-3}b^{-6}}$$

$$= \frac{5^{-3} \times 2^2 \times a^6 \times a^6 \times b^{-3} \times b^{-2}}{2 \times 5^2 \times a^2 \times a^{-3} \times b^3 \times b^{-6}}$$

$$= \frac{5^{-3} \times 2^2 \times a^{6+6} \times b^{-3+(-2)}}{5^{-2} \times 2 \times a^{2+(-3)} \times b^{3+(-6)}}$$

$$= \frac{5^{-3} \times 2^2 \times a^{12} \times b^{-5}}{5^{-2} \times 2 \times a^{-1} \times b^{-3}}$$

$$= 5^{-3-(-2)} \times 2^{2-1} \times a^{12-(-1)} \times b^{-5-(-3)}$$

$$= 5^{-1} \times 2 \times a^{13} \times b^{-2}$$

$$= \frac{1}{5} \times 2 \times a^{13} \times \frac{1}{b^2}$$

$$= \frac{2a^{13}}{5b^2}$$

$$(a^m)^n = a^{mn}$$

simplify powers

collect like terms

$$a^m \times a^n = a^{m+n}$$

simplify powers

$$\frac{a^m}{a^n} = a^{m-n}$$

simplify powers

$$a^{-m} = \frac{1}{a^m}$$

$$b) \frac{2x^4 y^{-5}}{3y^6 x^{-2}} \times \left( \frac{4xy^{-2}}{3x^6 y^3} \right)^{-3}$$

$$= \frac{2x^4 y^{-5}}{3y^6 x^{-2}} \times \frac{(2^2)^{-3} \times x^{1 \times -3} \times y^{-2 \times -3}}{3^{1 \times -3} \times x^{-6 \times -3} \times y^{3 \times -3}}$$

$$(a^m)^n = a^{mn}$$

$$= \frac{2x^4 y^{-5}}{3y^6 x^{-2}} \times \frac{2^{-6} \times x^{-3} \times y^6}{3^{-3} \times x^{18} \times y^{-9}}$$

simplify powers

$$= \frac{2^1 \times 2^6 \times x^4 \times x^{-3} \times y^{-5} \times y^6}{3^1 \times 3^{-3} \times x^{-2} \times x^{18} \times y^6 \times y^{-9}}$$

collect like terms

$$= \frac{2^{1+6} \times x^{4+(-3)} \times y^{-5+6}}{3^{1+(-3)} \times x^{-2+18} \times y^{6+(-9)}}$$

$$a^m \times a^n = a^{m+n}$$

$$= \frac{2^7 \times x^1 \times y^1}{3^{-2} \times x^{16} \times y^{-3}}$$

simplify powers

$$= 2^{-5} \times \frac{1}{3^{-2}} \times x^{1-16} \times y^{1-(-3)}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$= \frac{1}{2^5} \times (3^{-2})^{-1} \times x^{-15} \times y^2$$

$$\frac{1}{a^m} = a^{-m}$$

$$= \frac{1}{2^5} \times 3^{-2 \times -1} \times \frac{1}{2^{15}} \times y^2$$

$$(a^m)^n = a^{mn}$$

$$= \frac{1}{2^5} \times 3^2 \times \frac{1}{2^{15}} \times y^2$$

$$= \frac{3^2 y^2}{2^5 \times 2^{15}}$$

$$= \frac{9y^2}{32 \times 2^{15}}$$

$$c) \left( \frac{2m^3n^4}{5m^{\frac{1}{2}}n} \right)^{\frac{1}{3}} \div \left( \frac{4m^{\frac{1}{2}}n^2}{5^{\frac{2}{3}}} \right)^{-\frac{1}{2}}$$

$$(a^m)^n = a^{m \cdot n}$$

$$= \frac{2^{1 \times \frac{1}{3}} \times m^{3 \times \frac{1}{3}} \times n^{4 \times \frac{1}{3}}}{5^{1 \times \frac{1}{3}} \times m^{\frac{1}{2} \times \frac{1}{3}} \times n^{1 \times \frac{1}{3}}} \div \frac{(2^2)^{-\frac{1}{2}} \times m^{\frac{1}{2} \times -\frac{1}{2}} \times n^{-2 \times -\frac{1}{2}}}{5^{-\frac{2}{3} \times \frac{1}{2}}}$$

simplify powers

$$= \frac{2^{\frac{1}{3}} \times m^1 \times n^{\frac{4}{3}}}{5^{\frac{1}{3}} \times m^{\frac{1}{6}} \times n^{\frac{1}{3}}} \div \frac{2^{-1} \times m^{-\frac{1}{6}} \times n^1}{5^{\frac{1}{3}}}$$

$$\frac{a}{b} = a \times \frac{1}{b}$$

$$= \frac{2^{\frac{1}{3}} \times m^1 \times n^{\frac{4}{3}}}{5^{\frac{1}{3}} \times m^{\frac{1}{6}} \times n^{\frac{1}{3}}} \times \frac{5^{\frac{1}{3}}}{2^{-1} \times m^{-\frac{1}{6}} \times n^1}$$

collect like terms

$$= \frac{2^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times m^1 \times n^{\frac{4}{3}}}{2^{-1} \times 5^{\frac{1}{3}} \times m^{\frac{1}{6}} \times m^{-\frac{1}{6}} \times n^{\frac{1}{3}} \times n^1}$$

$$a^m \times a^n = a^{m+n}$$

$$= \frac{2^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times m^1 \times n^{\frac{4}{3}}}{2^{-1} \times 5^{\frac{1}{3}} \times m^{\frac{1}{6} + (-\frac{1}{6})} \times n^{\frac{1}{3} + 1}}$$

simplify powers

$$= \frac{2^{\frac{1}{3}} \times 5^{\frac{1}{3}} \times m^1 \times n^{\frac{4}{3}}}{2^{-1} \times 5^{\frac{1}{3}} \times m^0 \times n^{\frac{4}{3} + 1}}$$

$$\frac{1}{3} + 1 = \frac{1}{3} + \frac{3}{3}$$

$$= 2^{\frac{1}{3} - (-1)} \times 5^{\frac{1}{3} - \frac{1}{3}} \times m^{1-0} \times n^{\frac{4}{3} - \frac{4}{3}}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$= 2^{\frac{1}{3} + 1} \times 5^0 \times m^1 \times n^0$$

simplify powers

$$= 2^{\frac{4}{3}} \times 1 \times m \times 1$$

$$a^0 = 1$$

$$= 2^{\frac{4}{3}} m$$

$$\begin{aligned}
 (18) a) \quad & (3 \times 5^6)^{\frac{1}{2}} \times 3^{\frac{3}{2}} \times 5^{-2} + (3^6 \times 5^{\frac{-1}{2}})^0 \\
 & = 3^{1 \times \frac{1}{2}} \times 5^{6 \times \frac{1}{2}} \times 3^{\frac{3}{2}} \times 5^{-2} + 1 \\
 & = 3^{\frac{1}{2}} \times 5^3 \times 3^{\frac{3}{2}} \times 5^{-2} + 1 \\
 & = 3^{\frac{1}{2} + \frac{3}{2}} \times 5^{3 + (-2)} + 1 \\
 & = 3^{\frac{4}{2}} \times 5^1 + 1 \\
 & = 3^2 \times 5 + 1 \\
 & = 9 \times 5 + 1 \\
 & = 45 + 1 \\
 & = 46
 \end{aligned}$$

$(a^m)^n = a^{mn}$   
 $a^0 = 1$

$$\begin{aligned}
 b) \quad & (6 \times 3^{-2})^{-1} \div \frac{(3^{\frac{1}{2}} \times 6^{\frac{1}{3}})^6}{-6^2 \times (3^{-3})^0} \\
 & = 6^{1 \times (-1)} \times 3^{-2 \times (-1)} \div \frac{3^{\frac{1}{2} \times 6} \times 6^{\frac{1}{3} \times 6}}{-6^2 \times 1} \\
 & = 6^{-1} \times 3^2 \div \frac{3^3 \times 6^2}{-6^2} \\
 & = 6^{-1} \times 3^2 \times \frac{-6^2}{3^3 \times 6^2} \\
 & = \frac{6^{-1} \times 3^2 \times (-1 \times 6^2)}{3^3 \times 6^2} \\
 & = \frac{-1 \times 6^{-1+2} \times 3^2}{3^3 \times 6^2}
 \end{aligned}$$

$(a^m)^n = a^{mn}$     $a^0 = 1$

$$= \frac{-1 \times 6^1 \times 3^2}{3^3 \times 6^2}$$

$$= -1 \times 6^{1-2} \times 3^{2-3}$$

$$= -1 \times 6^{-1} \times 3^{-1}$$

$$= -1 \times \frac{1}{6^1} \times \frac{1}{3^1}$$

$$= -1 \times \frac{1}{6} \times \frac{1}{3}$$

$$= \frac{-1}{18}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

simplify powers.

### Problem Solving:

$$\textcircled{1} \quad \frac{6a^{3m} \times 2b^{2m} \times (3ab)^{-m}}{(4b)^m \times (9a^{4m})^{\frac{1}{2}}}$$

$$= \frac{6a^{3m} \times 2b^{2m} \times 3^{-m} \times a^{-m} \times b^{-m}}{4^m \times b^m \times (3^2)^{\frac{1}{2}} \times a^{4m \times \frac{1}{2}}}$$

$$= \frac{2^1 \times 2^1 \times 3^{-m} \times a^{3m} \times a^{-m} \times b^{2m} \times b^{-m}}{4^m \times 3^1 \times a^{2m} \times b^m}$$

$$= \frac{2^2 \times 3^{-m} \times a^{3m+(-m)} \times b^{2m+(-m)}}{2^{2m} \times a^{2m} \times b^m}$$

$$= \frac{2^2 \times 3^{-m} \times a^{2m} \times b^m}{2^{2m} \times a^{2m} \times b^m}$$

substitute in  $m=2$ .

$$\begin{aligned} \frac{2^2 \times 3^{-2}}{2^{2(2)}} &= 2^{2-4} \times 3^{-2} \\ &= 2^{-2} \times 3^{-2} \\ &= \frac{1}{2^2} \times \frac{1}{3^2} \\ &= \frac{1}{4} \times \frac{1}{9} \\ &= \frac{1}{36}. \end{aligned}$$

② Investigate the powers of 3, 6 and 8 and look for patterns

a)

$3^1 =$	03
$3^2 =$	09
$3^3 =$	27
$3^4 =$	81
$3^5 =$	243
	... 29
	87
	61
	83
	...
	...

b)

$6^1 =$	6
$6^2 =$	36
$6^3 =$	216

each time the one's digit (6) is multiplied by 6 to give 36.

the ones digit is always 6!

c)

$8^1 =$	8
$8^2 =$	64
$8^3 =$	512
$8^4 =$	4096
$8^5 =$	32768
$8^6 =$	... 4
$8^7 =$	... 2
$8^8 =$	... 6
$8^9 =$	... 8

the pattern repeats every 4.

$8^{1007} \Rightarrow 8^7$   
 $\therefore$  the ones digit is 2.

$3^{3^2} = 3^{27}$

the pattern begins to repeat at  $3^4$   
 $\therefore 3^{3^2}$  has the same 10's unit as  $3^7$  which is 8.